Uniform Electric Field

Objectives

After going through this lesson, the learners will be able to:

- Understand the nature of electric field
- Distinguish between uniform and non-uniform electric field
- Draw field lines due to different sources of electric field.
- Explain the motion of a point charge in a uniform electric field.
- Apply the principle of superposition of point charges to solve problems based on electric field due to different charge configurations

Content Outline

- Unit syllabus
- Module wise distribution of unit syllabus
- Words you must know
- Introduction
- Concept of electric field E
- Physical Significance of electric field
- Electric field lines another way of describing electric field
- Principle of superposition for electric field
- Uniform and non-uniform electric field
- Numerical problems
- Summary

Unit Syllabus

Chapter-1: Electric Charges and Fields

Electric Charges; Conservation of charge, Coulomb's law- force between two point charges, forces between multiple charges; superposition principle and continuous charge distribution.

Electric field; electric field due to a point charge; electric field lines; electric dipole; electric field due to a dipole; torque on a dipole in uniform electric field.

Electric flux, statement of Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside).

Chapter-2: Electrostatic Potential and Capacitance

Electric potential, potential difference, electric potential due to a point charge, a dipole and system of charges; equipotential surfaces, electrical potential energy of a system of two point charges and of electric dipole in an electrostatic field.

Conductors and insulators, free charges and bound charges inside a conductor. Dielectrics and electric polarization, capacitors and capacitance, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor.

Module Wise Distribution

Module 1	• Electric charge
	Properties of charge
	Coulombs' law
	Characteristics of coulomb force
	• Constant of proportionality and the intervening medium
	Numerical Examples
Module 2	Forces between multiple charges
	Principle of superposition
	Continuous distribution of charges
	• Numerical examples
Module 3	Electric field E
	• Importance of field E and ways of describing field
	Point charges superposition of electric field
	• Numerical examples
Module 4	Electric dipole
	• Electric field of a dipole
	Charges in external field
	• Dipole in external field Uniform and non-uniform
Module 5	Electric Flux
	• Flux density
	• Gauss theorem
	• Application of gauss theorem to find electric field
	• For a distribution of charges

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Words You Must Know

Let us recollect the words we have been using in our study of this physics course.

- Electric Charge: Electric charge is an intrinsic characteristic of many of the fundamental particles of matter that gives rise to all electric and magnetic forces and interactions.
- **Conductors:** Some substances readily allow passage of electricity through them, others do not. Those which allow electricity to pass through them easily are called

conductors. They have electric charges (electrons) that are comparatively free to move inside the material. Metals, earth, human and animal bodies are all conductors of electricity.

- **Insulators:** Most of the non-metals, like glass, porcelain, plastic, nylon, wood, offer high opposition to the passage of electricity through them. They are called *insulators*.
- **Point Charge:** When the linear size of charged bodies is much smaller than the distance separating them, the size may be ignored and the charge bodies can then be treated as *point charges*.
- **Conduction:** Transfer of electrons from one body to another. It also refers to flow of charges electrons in metals and ions in electrolytes and gases
- Induction: The temporary separation of charges in a body due to a charged body in the vicinity. The effect lasts as long as the charged body is held close to the body in which induction is taking place
- Quantization of charges: Charge exists as an integral multiple of basic electronic charge. Charge on an electron is $1.6 \times 10^{-19}C$
- Electroscope: A device to detect charge, to know the relative magnitude of charge on two charged bodies. A suitably charged electroscope can also detect the kind of charge (positive or negative)
- **Coulomb:** S.I unit of charge defined in terms of 1 ampere current flowing in a wire to be due to 1 coulomb of charge flowing in 1 s

1 coulomb = collective charge of 6×10^{18} electrons

- **Conservation of charge:** Charge can neither be created or destroyed in an isolated system it(electrons) only transfers from one body to another.
- **Coulomb's Force:** It is the electrostatic force of interaction between the two point charges.

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

• Vector form of Coulomb's law: A mathematical expression based on coulomb's law to show the magnitude as well as direction of mutual electrostatic force between two or more charges

$$F_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} r^{\Lambda} r_{12}$$

• Laws of vector addition:

Triangle law of vector addition: If two vectors are represented by two sides of a triangle in order, then the third side represents the resultant of the two vectors Parallelogram law of vector addition: If two vectors are represented in magnitude and direction by adjacent sides of a parallelogram then the resultant of the vectors is given by the diagonal passing through their common point

Also resultant of vectors P and Q acting at angle of θ is given by

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

Polygon law of vector addition: Multiple vectors may be added by placing them in order of a multi sided polygon; the resultant is given by the closing side taken in opposite order.

Resolution of vectors into components and then adding along x, y and z directions

- Linear charge density: The *linear charge density*, *λ* is defined as the charge per unit length.
- Surface charge density: The surface *charge density* σ is defined as the charge per unit surface area
- Volume charge density: The volume charge density ρ is defined as the charge per unit volume.
- Superposition Principle: For an assembly of charges $q_1, q_2, q_3, ...$, the force on any charge, say q_1 , is the vector sum of the force on q_1 due to q_2 , the force on q_1 due to q_3 , and so on. For each pair, the force is given by Coulomb's law for two point charges.

Introduction

Let us consider a point charge Q placed in vacuum, at the origin (0, 0, 0). If we place another point charge q at a point P say (x, y, z) then we know that the charge Q will exert a force on qas per Coulomb's law. We may ask the question: If charge q is removed, then what is left in the surrounding? Is there nothing? If there is nothing at the point P, then how does a force act when we place the charge q at P?

In order to answer such questions, the early scientists introduced the concept of *field*. According to this, we say that the charge Q produces an electric field everywhere in the surrounding area. In three-dimensional space, this may be imagined as a region of influence around a charge. When another charge q is brought at some point P within the field a force acts on the charge.

Effectively the charge has a region of influence around it; this is indicated by the force (as per coulomb's law) experienced by a charge at different locations within the influenced space. Beyond the region of influence, the electrostatic force is not experienced by any small charge.

Concept of Electric Field 'e'

Let us consider an electric charge Q located in a space. If we bring another charge q near the charge Q, then q experiences a force of attraction or repulsion (interaction), due to Q. The force experienced by q is said to be the "Electric field" set up by the charge Q.

Thus, the space surrounding an electric charge Q, in which another charge q experiences an electrostatic force of interaction, is called the electric field of Q.

The charge Q is called the "source charge" and the charge q is called the "test charge".

The source here may be a point charge, a group of point charges or a continuous distribution of charges.

Source and test charge: The charge, which is producing the electric field, is called a *source charge* and the charge, which tests the effect of a source charge, is called a *test charge*.

Also test charge is vanishingly small, so that it does not change the relative position of source, although the forces are equal and opposite. Also the test charge should not change the strength of the overall field.

(Note that the charge *q* also exerts an equal and opposite force on the Charge *Q*). Electric Field Intensity or Strength

To determine the magnitude of an electric field at a point, we place a test charge at that point and measure the electrostatic force acting at that point per unit charge. Hence the electric field intensity E at a point is the ratio of the force acting on test charge placed at that point to the magnitude of the test charge. Electric field strength is <u>a vector quantity</u>; it has both magnitude and direction.

The magnitude of the electric field strength is simply defined as the force per unit charge on the test charge.

$$Electric Field strength = \frac{Force}{Charge}$$

If the electric field strength is denoted by the symbol **E**, then the equation can be rewritten in symbolic form as:

The electric field produced by the point charge
$$Q$$
 placed in vacuum at a point located at a distance r is given as:

$$E(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r}$$

 $E = \frac{F}{q}$

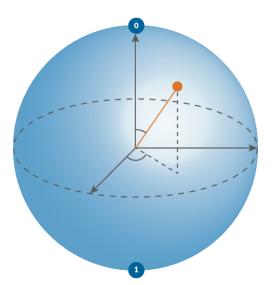
position of the point being considered, where, $r^{r} = |r|/r$, is a unit vector from the origin to the point at distance r.

Thus, above equation specifies the value of the electric field for each value of the position vector \mathbf{r}

The magnitude or strength of the electric field at a point depends upon

- Magnitude of charge
- Distance of the point (test charge) from the source charge
- Spatial location of the point around the source charge

Think About This



https://www.sciencenews.org/article/quarter-century-ago-qubit-was-born

A positive charge Q is located at the center of the sphere of radius r

Consider the electrostatic force on a test charge at 0 and 1

- Will the magnitude of force be the same?
- Will the force be the same?
- What is the difference?
- What is the electric field at 1 and 0?
- What is the electric field at the location of the red dot?
- What is the electric field at the foot of the perpendicular? (shown by dotted line)
- What if the charge Q is negative?

Unit of electric field strength

The S.I. units of electric field strength E arises from its definition. Since an electric field is defined as a force per unit charge, its unit would be a force unit divided by charge unit.

That is $\frac{newton}{coloumb}$ or N/C. or NC^{-1}

We can also infer from the above equation that if q is unity, the electric field due to a source charge Q is numerically equal to the force exerted by it.

Thus, we can also say the electric field due to a charge Q at a point in space may be defined as the force that a unit positive charge would experience if placed at that point. Considering positive charge is only a convention.

Electric Charge and Field at a point

• Note that the source charge Q must remain at its original location. However, if a charge q is brought at any point around Q, Q itself is bound to experience an electrical force due to q and will tend to move. A way out of this difficulty is to make q negligibly small.

The force F is then negligibly small but the ratio F/q is finite and defines the electric field. That is, E equals to:

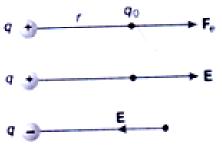
$$= \frac{F}{q}$$

A practical way to get around the problem (of keeping Q undisturbed in the presence of (q) is to hold Q to its location by unspecified forces.

This may look strange but actually this is what happens in practice.

For exampleWhen we are considering the electric force on a test charge q due to a

charged planar sheet, the charges on the sheet are held to their locations by the forces due to the unspecified charged constituents inside the sheet.



• Although the electric field E due to source charges Q is defined in terms of test charge q, it is independent of magnitude of q.

This is because F is proportional to magnitude q, so the ratio F/q does not depend on q.

Hence the electric field due to the source charge Q is independent of its magnitude but depends on the particular location of test charge q which may take any value in the space around the source charge Q.

Thus, the electric field E due to Q is also dependent on the space coordinate r. For different positions of the test charge q all over the space, we get different values of electric field E. The field exists at every point in three-dimensional space.

- For a positive charge, the electric field vector will be directed radially outwards from the charge. On the other hand, if the source charge is negative, the electric field vector, at each point, points radially inwards.
- Since the magnitude of the force F on charge q due to charge Q depends only on the distance r of the charge q from charge Q, the magnitude of the electric field E will also depend only on the distance r. Thus at equal distances from the charge Q, the magnitude of its electric field E is the same.

The magnitude of electric field E due to a point charge is thus the same on a sphere with point charge at the center c in other words, it has a spherical symmetry.

• If we denote the position of charge q by the vector r, it experiences a force F equal to the charge q multiplied by the electric field E, at the location of q. Thus

$$F(r) = q E(r).$$

This Equation signifies that if we know the field but not the source we can still calculate the force it exerts on the q

Think About These

- What if the source charge is not a single point charge but a finite metal spherical charge, what will be the electric field? How will you describe it in terms of the above?
- What if there was a collection of charges? Say made up of a bunch of inflated balloons which were getting charged due to mutual shuffling?

• What if there is a flexi banner which is getting charged due to people rubbing their hands on it?

A working TV screen, or computer monitor gets charged. You can check this by taking thin paper strips, iron them lightly and take them close to it.

What kind of electrical influence would the above create around them?

Can we extend the idea that an electric field will exist around all charged systems? But in our course we will take very simple situations e.g. take point charge, consider collection of point charges, and take plane sheets of charged bodies for which it is geometrically easy for us to imagine the field.

You can try this

Take a small plastic bag. Close the mouth using a string. Cut the rest of the bag into strips. Stroke the cut section several times. Hold the bag up by the string. The striped fan out. Think why?

Physical Significance of Electric Field

You may wonder why the notion of electric field has been introduced here at all. After all, for any system of charges, the measurable quantity is the force on a charge which can be directly determined using Coulomb's law and the superposition principle.

Why then introduce this intermediate quantity called the electric field?

For electrostatics, the concept of electric field is convenient, but not really necessary.

Electric field is an elegant way of characterizing the electrical environment of a system of charges. Electric field at a point in the space around a system of charges tells you about the force a unit positive test charge would experience if placed at that point (without disturbing the system).

Electric field is a characteristic of the system of charges and is independent of the test charge that you place at a point to determine the field.

The term *field* in Physics generally refers to a quantity that is defined at every point in space and may vary from point to point. Electric field is a vector quantity, since force is a vector quantity.

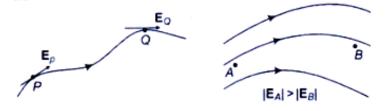
Only for information

The true physical significance of the concept of electric field, however, emerges only when we go beyond electrostatics and deal with time dependent electromagnetic phenomena. Suppose we consider the force between two distant charges q_1 , q_2 in accelerated motion. Now the greatest speed with which a signal or information can go from one point to another is c, the speed of light. Thus, the effect of any motion of q_1 on q_2 cannot arise instantaneously. There will be some time delay between the effect (force on q_2) and the cause (motion of q_1). It is precisely here that the notion of electric field (strictly, electromagnetic field) is natural and very useful. *he accelerated motion of charge* q_1 *produces electromagnetic waves, which then propagate with the speed* c, *reach* q_2 *and cause a force on* q_2 . The notion of field elegantly accounts for the time delay. Thus, even though electric and magnetic fields can be detected only by their effects (forces) on charges, they are regarded as physical entities, not merely mathematical constructs. They have an *independent dynamic* of their own, i.e., they evolve according to laws of their *own*. They can also transport energy. Thus, a source of time dependent electromagnetic fields, turned on briefly and switched off, leaves behind propagating electromagnetic fields transporting energy. The concept of field was first introduced by Faraday and is now among the central concepts in physics.

Electric Field Lines- Another Way of Describing Electric Field

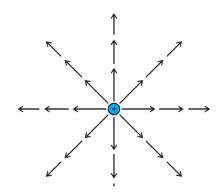
As we have seen, electric charges create an electric field in the space surrounding them. The field strength at a point is calculated in terms of force per unit charge. It is useful to have a kind of map that gives the direction and indicates the strength of the field at various places. **Field lines**, a concept introduced by Michael Faraday, provide us with an easy way to visualize the electric field.

"An electric field line is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric field vector at that point. The relative closeness of the lines at some places gives an idea about the intensity of the electric field at that point."



Let us try to represent E due to a point charge pictorially.

Let the point charge be placed at the origin.



Field of a point charge

Draw vectors pointing along the direction of the electric field

Make lengths of the vectors proportional to the strength of the field at each point. Since the magnitude of the electric field at a point decreases inversely as the square of the distance of that point from the charge, the vector gets shorter as one goes away from the origin, always pointing radially outward.

In this figure, each arrow indicates the electric field, i.e., the force acting on a unit positive charge, placed at the tail of that arrow.

Connect the arrows pointing in one direction and the resulting figure represents a field line.

We thus get many field lines, all pointing outwards from the positive point charge

This is because our test charge was positive; if the source charge is negative the lines would point towards it as the force on the test charge would be attractive.

Have we lost the information about the strength or magnitude of the field now, because it was contained in the length of the arrow? No.

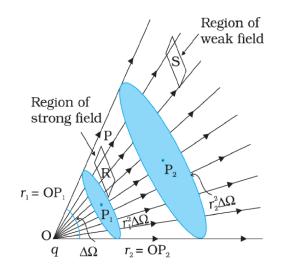
Now the magnitude of the field is represented by the density of field lines. (number of lines enclosed in a certain space)

E is strong near the charge, so the density of field lines is more near the charge and the lines are closer.

Away from the charge, **the field gets weaker and the density of field lines is less**, resulting in well-separated lines. Another person may draw more lines.

But the number of lines is not important. In fact, an infinite number of lines can be drawn in any region.

It is the relative density of lines in different regions which is more important.



Note

We draw the figure on the plane of paper, *i.e.*, in two dimensions but we live in three-dimensions. So if one wishes to estimate the density of field lines, one has to consider the number of lines per unit cross-sectional area, perpendicular to the lines. Since the electric field decreases as the square of the distance from a point charge and the area enclosing the charge increases as the square of the distance, the number of field lines crossing the enclosing area remains constant, whatever may be the distance of the area from the charge.

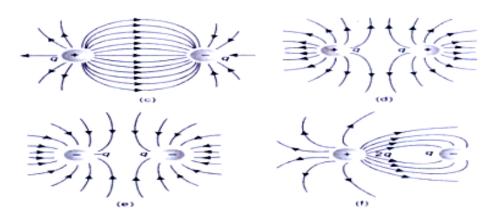
Dependence of electric field strength on the distance and its relation to the number of field lines.

The picture of field lines was invented by Faraday to develop an intuitive non- mathematical way of visualizing electric fields around charged configurations. Faraday called them *lines of force*. *This term is somewhat misleading, especially in the case of magnetic fields. The more appropriate term is field lines.*



As mentioned earlier, the field lines are in 3-dimensional space, though the figure shows them only in a plane. The field lines of a single positive charge are radially outward while those of a single negative charge are radially inward. The field lines around a system of two positive charges (q, q) give a vivid pictorial description of their mutual repulsion, while those around the configuration of two equal and opposite charges

(q, -q), a dipole, shows clearly the mutual attraction / repulsion between the charges.



Field lines due to some simple charge configurations.

The field lines follow some important general properties

- Electric field lines always begin on a positive charge and end on a negative charge and do not start or stop in mid space.
- The tangent to a line at any point gives the direction of E at that point. This is also the path on which a positive test charge will tend to move if free to do so.
- Two lines can never intersect. If it happens then two tangents can be drawn at their point of intersection, *i.e.*, intensity at that point will have two directions which are absurd.
- In a uniform field, the field lines are straight parallel and uniformly spaced.
- The electric field lines can never form closed loops as a line can never start and end on the same charge.
- Electric field lines also give us an indication of the equipotential surface (surface which has the same potential)
- Electric field lines always flow from higher potential to lower potential.
- In a region where there is no electric field, lines are absent. This is why inside a conductor (where the electric field is zero) there cannot be any electric field line.
- Electric lines of force end or start normally from the surface of a conductor.

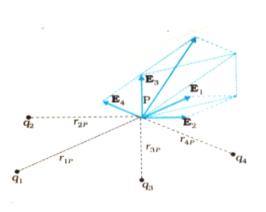
Principle of Superposition of Electric Field

Experimentally, it is verified that force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges. This is termed as the principle of superposition

Combined field at a point in space due to a collection of point charges

If there are *n* point charges in a space $q_1 q_2 q_3 \dots q_n$, then each of them will produce the same intensity at any point which it would have produced in the absence of other point charges. Hence electric field intensity E at any point P due to all n point charges will be equal to the vector sum of electric field intensities $E_1, E_2, E_3, \dots E_n$ produced by individual charges at the point P. Hence

 $E = E_1 + E_2 + \dots + E_n$



Electric field at a point due to a system of charges is the vector sum of the electric fields at the point due to individual charges.

For continuous charge distribution

If there is a continuous charge distribution, then the field due to continuous charge distribution can be obtained in much the same way as for a system of discrete charges. Suppose a continuous charge distribution in space has a volume **charge density** (ρ). Choose any convenient origin O and let the position vector of any point in the charge distribution be r. Divide the charge distribution into small volume elements of size ΔV .

The charge in a volume element ΔV will be $\rho \Delta V$.

$$\Delta E = \frac{1}{4\pi\varepsilon_0} \frac{\rho\Delta V}{r} r'$$

Now, consider any general point P(inside or outside the distribution) with position vector **r**. Electric field due to the charge $\rho\Delta V$ is given Coulomb's law where's the distance between the charge element P and $\hat{}$ is a unit vector in the direction from the charge element to P. By the superposition principle, the total electric field due to the charge distribution is obtained by summing over electric fields due to different volume elements or integrating the same.

$$E = \frac{1}{4\pi\varepsilon_0} \sum_{all \,\Delta V} \frac{\rho \Delta V}{r} \hat{r}$$

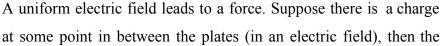
Uniform And Non-Uniform Field

Now we know that the electric field of a point charge depends upon the location of the charge. Hence has different magnitude and direction at different points. We refer to this field as a non-uniform electric field.

For the practical purpose of studying the motion of charge in an electric field, we define a uniform field. A uniform electric field is one whose magnitude and direction is same at all points in space and it will exert same force of a charge regardless of

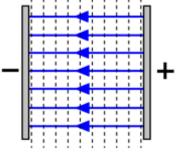
the position of charge.

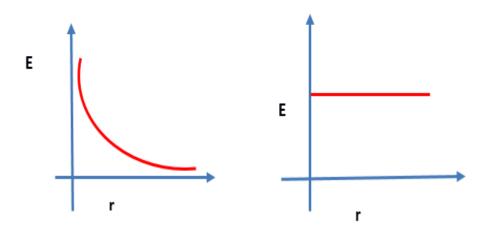
A uniform electric field is set up by two straight parallel metal plates that have equal and opposite charge. The direction of the field is from positive charge to negative charge.



electrostatic force acting on a charge +q is qE in the direction of E, while on the charge -q it is qE in the opposite direction of E.

Graphically





The first graph shows E proportional to $1/r^2$: suggesting non uniform field The second graph shows E independent of r : suggesting uniform field.

Numerical Examples For Better Understanding

Example:

There is an electric field of 10^5 N/C points due west at a certain spot. Find the magnitude and direction of the force that acts on a charge of + $2\mu C$ and -5 μC at this spot?

Solution:

Force on + $2\mu C$ = q E =2 ×10⁻⁶ ×10⁵ = 0.2N (due west)

Force on -5 μC = 5 ×10^{-6×} 10⁵ = 0.5N (due east)

Example:

Two positive point charges $q_1 = 16 \ \mu\text{C}$ and $q_2 = 4 \ \mu\text{C}$ are separated in vacuum by a distance of 3.0 m. Find the point on the line between the charges where the net electric field is zero.

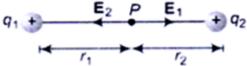
Solution:

Between the charges, the two field contributions have opposite directions, and the net electric field is zero at a point (say P), where the magnitudes of E_1 and E_2 are equal.

However, since $q_2 q_1$, point P must be closer to q_2 , in order that the field of the smaller charge can balance the field of the larger charge.

At P,
$$E_1 = E_2$$

Or,
 $\frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1^2} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2^2}$
 $\therefore \frac{r_1}{r_2} = \sqrt{\frac{q_1}{q_2}} = \sqrt{\frac{16}{4}} = 2$



Also, $r_1 + r_2 = 3.0 \text{ m}$

Solving these equations, we get:

 $r_1 = 2 m and r_2 = 1 m$

Thus, the point P is at a distance of 2 m from q_1 and 1 m from q_2 .

We must note that the point in the space where the net field due to the system of point charges or continuous **charge distribution is zero is called "Neutral Point"**.

Thus, in the above example point P is the neutral point of the system.

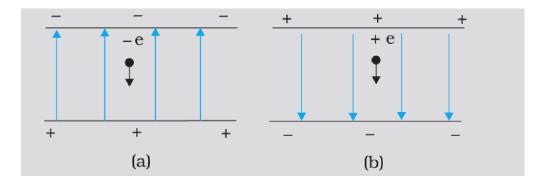
Example:

An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude 2.0×10^4 N C⁻¹ Fig (a). The direction of the field is reversed keeping its magnitude unchanged and a proton falls through the same distance Fig (b)

Compute

The time of fall in each case:

Contrast the situation with that of 'free fall under gravity'.



Solution

In Fig. (a) The field is upward, so the negatively charged electron experiences a downward force of magnitude eE, where E is the magnitude of the electric field.

The acceleration of the electron is: $a_e = \frac{eE}{m_e}$, where m_e is the mass of the electron.

Starting from rest, the time required by the electron to fall through distance h is given by:

$$t_e = \sqrt{\frac{2h}{a_e}} = \sqrt{\frac{2hm_e}{eE}}$$

For $e = 1.6 \times 10^{-19} \text{ C}$, $m_e = 9.11 \times 10^{-31} \text{ Kg}$ $E = 2.0 \times 10^4 \text{ N C}^{-1}$, $h = 1.5 \times 10^2 \text{ m}$ $t_e = 2.9 \times 10^{-9}$

In Fig. (b), the field is downward, and the positively charged proton experiences a downward force of magnitude eE. The acceleration of the proton is:

$$a_p = \frac{eE}{m_p}$$

Where m_p is the mass of the proton; $m_p = 1.67 \times 10^{-27}$ kg. The time of fall for the proton is:

$$t_p = \sqrt{\frac{2h}{a_p}} = \sqrt{\frac{2hm_p}{eE}} = 1.3 \times 10^{-7} s$$

Thus, the heavier particle (proton) takes a greater time to fall through the same distance.

This is in basic contrast to the situation of 'free fall under gravity' where the time of fall is independent of the mass of the body. Note that in this example we have ignored the acceleration due to gravity in calculating the time of fall. To see if this is justified, let us calculate the acceleration of the proton in the given electric field:

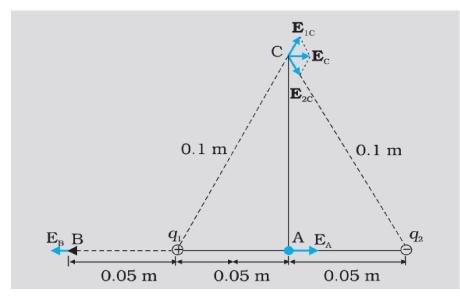
$$a_p = \frac{eE}{m_p}$$

$$= \frac{(1.6 \times 10^{-19} C) \times (2.0 \times 10^{-1} N C^{-1})}{1.67 \times 10^{-27} kg}$$
$$= 1.9 \times 10^{12} \text{ ms}^{-2}$$

Which is enormous compared to the value of g (9.8 ms⁻²), the acceleration due to gravity. The acceleration of the electron is even greater. Thus, the effect of acceleration due to gravity can be ignored in this example.

Example:

Two point charges q_1 and q_2 , of magnitude +10⁻⁸ C and -10⁻⁸ C respectively are placed 0.1 m apart. Calculate the electric fields at points A, B and C shown in Fig.



Solution:

The electric field vector \mathbf{E}_{1A} (not marked in the fig) at A due to the positive charge q_1 points towards the right and has a magnitude.

$$E_{1A} = \frac{(9 \times 10^{9} Nm^{2} C^{-2}) \times (10^{-8} C)}{(0.05 m)^{2}} = 3.6 \times 10^{4} NC^{-1}$$

The electric field vector \mathbf{E}_{2A} at A due to the negative charge q_2 points towards the right and has the same magnitude. Hence the magnitude of the total electric field \mathbf{E}_A at A is:

$$E_A = E_{1A} + E_{2A} = 7.2 \times 10^4 N C^{-1}$$

E_A is directed toward the right.

The electric field vector \mathbf{E}_{1B} at B due to the positive charge q_1 points towards the left and has a magnitude:

$$E_{1B} = \frac{(9 \times 10^{9} Nm^{2} C^{-2}) \times (10^{-8} C)}{(0.05 m)^{2}} = 3.6 \times 10^{4} N C^{-1}$$

The electric field vector E_{2B} at B due to the negative charge q_2 points towards the right and has a magnitude:

$$E_{2B} = \frac{(9 \times 10^9 Nm^2 C^{-2}) \times (10^{-8} C)}{(0.15 m)^2} = 4 \times 10^3 N C^{-1}$$

The magnitude of the total electric field at B is:

$$E_{B} = E_{1B} - E_{2B} = 3.2 \times 10^{4} NC^{-1}$$

 E_B is directed towards the left.

The magnitude of each electric field vector at point C, due to charge q_1 and q_2 is:

$$E_{1C} = E_{2C} = \frac{(9 \times 10^9 Nm^2 C^{-2}) \times (10^{-8} C)}{(0.10 m)^2} = 9 \times 10^3 N C^{-1}$$

The directions in which these two vectors point are indicated in Fig. The resultant of these two vectors is:

$$E_{c} = E_{1c} \cos \frac{\pi}{3} + E_{2c} \cos \frac{\pi}{3} = 9 \times 10^{3} N C^{-1}$$

E_c points towards the right.

Summary

- The electric field E at a point due to a charge configuration is the force on a small positive test charge q placed at the point divided by the magnitude of the charge. Electric field due to a point charge q has a magnitude $|q|/4\pi\epsilon_0 r^2$; it is radially outwards from q, if q is positive and radially inwards if q is negative. Like Coulomb force, the electric field also satisfies the superposition principle.
- An electric field line is a curve drawn in such a way that the tangent at each point on the curve gives the direction of the electric field at that point. The relative closeness of field

lines indicates the relative strength of electric field at different points; they crowd near each other in regions of strong electric field and are far apart where the electric field is weak. In regions of constant electric field, the field lines are uniformly spaced parallel straight lines

- Some of the important properties of field lines are:
 - Field lines are continuous curves without any breaks.
 - Two field lines cannot cross each other.
 - Electrostatic field lines start at positive charges and end at negative charges they cannot form closed loops.
- Electric field is added by the principle of superposition and uses vectors for the same.
- There are three ways to describe electric fields:
 - Force per unit charge at any point the test field
 - Number density of field lines per unit area considered normal to the lines
 - Graphical representation